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MODEL SIMILARITY FOR BUILDUP OF PORE PRESSURE AROUND A GRAVITY FOUNDATION UNDER WAVES

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1. Problem statement

A gravity foundation sitting on the bed is subject to a progressive wave, Fig. 1. The dimensions of the structure are W, the width of the structure, and h_a, the height of the structure (anchor gravity foundation). We study the 2D case for simplicity. We will, however, discuss the 3D case later in the report. The wave properties are H, the wave height, T, the wave period, h, the water depth, and L, the wave length. The soil depth is d. The coordinate system is shown in Fig. 1, with depth z measured downwards from the seabed.

The seabed under a progressive wave undergoes periodic shear strains/stresses. If the soil grains are initially loosely packed, these cyclic shear strains/stresses gradually rearrange the soil grains at the expense of the pore volume of the soil. The latter effect pressurizes the water in the pores, and presumably leads up to a buildup of pore pressure if the soils is fine sand or silt (usually called undrained soils). As the wave action continues, the pore pressure will continue to accumulate. The accumulated pressure can even reach to such levels that the soil can be liquefied and become virtually a liquid, a process called liquefaction.

This report presents the results of a study, investigating the similarity criteria for physical modelling of the previously mentioned process, namely, the buildup of pore pressure in marine soils with the presence of a gravity foundation.

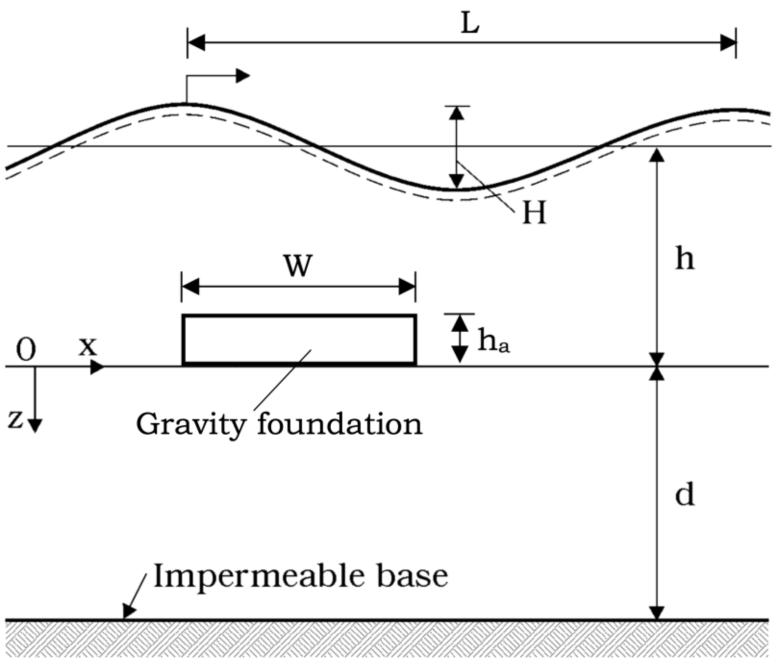

Fig. 1. Definition sketch.

2. Geometrical similarity

To achieve a true geometrical similarity in the model, all lengths associated with the prototype should, for the most part, be scaled down according to the length scale, N. (We use the same notation as in the book by Sumer, 2014, p. 98.) Therefore, the structure dimensions in the model will be

$$W_m = \frac{W_p}{N} \quad \text{and} \tag{1}$$

$$h_{am} = \frac{h_{ap}}{N} \tag{2}$$

Here and thereafter, the quantities with sub-index p and those with the sub-index m indicate the prototype and model values, respectively. The buildup of pore pressure around the structure is closely related with the "interplay" between the wave length, L, and the structure width, W. Therefore, the prototype wave length should also be scaled down with the same length scale, namely, N, for a true similarity between the prototype and the model:

$$L_m = \frac{L_p}{N} \tag{3}$$

in which L_p is the prototype wave length and L_m is the model wave length. We note that there is one more quantity with the dimension of length, namely the water depth, h in the process. We will return to this quantity later in the report.

3. Similarity concerning buildup of pore pressure

The nondimensional parameters governing the buildup of pore pressure in a progressive wave are as tabulated in Table 1 (Sumer, 2014, p. 102).

Table 1. Nondimensional parameters governing the buildup of pore pressure.

Nondimensional governing parameters:		
Time	ωt	(4)
Depth measured downward from the bed	λz	(5)
Soil depth	λd	(6)
Wave severity	χ_0	(7)
Relative density of the soil	D_r	(8)
Nondimensional length of diffusion of accumulated pressure	S	(9)

In Table 1, t is time, ω the angular frequency defined by

$$\omega = \frac{2\pi}{T} \quad , \tag{10}$$

λ the wave number defined by

$$\lambda = \frac{2\pi}{L} \quad , \tag{11}$$

χ_0 the so-called wave severity defined by

$$\chi_0 = \left(\frac{\tau}{\sigma'_{v0}} \right)_{z=0} = \frac{p_b \lambda}{\gamma'} \tag{12}$$

in which τ is the maximum shear stress (amplitude) in the soil in the $x-$direction, σ'_{v0} the initial vertical effective stress, γ' the submerged specific weight of the soil, and p_b the maximum value (the amplitude) of the pressure exerted on the bed by the wave, which is given by (Sumer, 2014, Eq. 2.46)

$$p_b = \gamma \frac{H}{2} \frac{1}{\cosh(\lambda h)} \tag{13}$$

with γ being the specific weight of water. We note that the quantity $\frac{\tau}{\sigma'_{v0}}$ is the familiar shear stress ratio in the diagram the shear stress ratio versus the number of cycles to cause liquefaction (see Sumer, 2024, Fig. 3.21), one of the nondimensional key parameters governing liquefaction.

Furthermore, the quantity D_r in Table 1 is the relative density of the soil defined by

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \quad (14)$$

in which e is the void ratio, and $e_{\max}$ the maximum void ratio and $e_{\min}$ the minimum void ratio. Finally, the quantity S in Table 1 is defined by

$$S = \frac{c_v T}{L^2} \quad (15)$$

in which c_v is the coefficient of consolidation of the soil.

4. Procedure for design of model experiments

The following procedure is to be used for the design of the model experiments, i.e., the design of the model structure, and the design of the wave conditions and that of the soil conditions in the model experiments.

4.1. Design of the model structure

First of all, select the model length scale, N.

Second, determine the dimensions of the model structure, W_m and h_{am}, using Eqs. (1) and (2).

4.2. Design of the model wave

Determine the model wave properties, namely the model wave period T_m, the model wave height H_m, the model wave length L_m, and the model water depth h_m, considered to be a wave parameter since this quantity involves in the determination of the wave period.

First, start off with the wave length L_m; Determine L_m, using Eq. (3).

Second, select a value for the water depth in the model, h_m, considering the height of the model structure determined in Step 2 above, h_{am}, and also the depth of the flume to be used in the model experiments. In this way, the selected model water depth may not necessarily satisfy the geometrical similarity, discussed in Section 2. This is not a problem, however, as it will not affect directly the model similarity concerning the buildup of pore pressure discussed in Section 3. The model similarity concerning the water depth will be taken care of through the similitude condition implied by Eq. (7) as will be shown shortly.

Third, given L_m and h_m, determine the model wave period T_m, using the dispersion relation for the model conditions, namely,

$$\omega_m^{\ 2} = g\lambda_m \tanh(\lambda_m h_m) \quad \text{with} \tag{16}$$

$$\lambda_m = \frac{2\pi}{L_m} \quad \text{and} \tag{17}$$

$$\omega_m = \frac{2\pi}{T_m} \tag{18}$$

and g being the acceleration due to gravity.

Fourth, given the prototype wave length and the prototype water depth, L_p and h_p, respectively, and, again, given the model wave length and the model water depth, L_m and h_m, respectively, and given the prototype wave height H_p, determine the model wave height H_m as follows.

To this end, we return to the nondimensional governing parameters, the parameters governing the process of buildup of pore pressure, tabulated in Table 1. For model similarity, these parameters should have the same values in both the prototype and the model, the underlying principle in the theory of model similarity. Of the six parameters in Table 1, we, for the purpose of determining the model wave height, resort to the parameter χ_0, Eq. (7). (We will return to the other parameters later in the report.) The parameter χ_0 is given by Eq. (12):

$$\chi_0 = \frac{p_b \lambda}{\gamma'} \qquad (19)$$

For model similarity, $\chi_{0p} = \chi_{0m}$ should be satisfied, or

$$\frac{p_{bp}\lambda_p}{\gamma'_p} = \frac{p_{bm}\lambda_m}{\gamma'_m} \qquad (20)$$

Now, provided that the same sediment is used in the model as in the prototype (also having the same void ratio), the denominators in the above equation will cancel out, and furthermore using Eq. (13), the above similarity condition, Eq. (20), will lead to

$$H_m = \frac{L_m}{L_p} \frac{\cosh(\frac{2\pi}{L_m} h_m)}{\cosh(\frac{2\pi}{L_p} h_p)} H_p \qquad (21)$$

This equation enables one to determine the model wave height H_m, given the quantities L_p, h_p, L_m, h_m, and H_p.

4.3. Design of the model soil

We, now, once again, return to the nondimensional governing parameters, the parameters governing the process of buildup of pore pressure, tabulated in Table 1. Of these parameters, three of them, namely the soil depth, Eq. (6), the relative density of soil, Eq. (8), and the nondimensional length of diffusion of accumulated pressure, Eq. (9), are related to the soil. Similarity principle requires that, here, too, these nondimensional quantities should have the same values in both the prototype and the model.

First of all, regarding the nondimensional soil depth, the following equation needs to be satisfied for model similarity:

$$\lambda_p d_p = \lambda_m d_m \qquad (22)$$

or

$$d_m = \frac{d_p}{N} \tag{23}$$

Therefore, given the prototype soil depth d_p, and the length scale N, determine the model soil depth from the above equation.

Second, the relative density of the soil D_r should be the same in both the prototype and the model, to achieve model similarity. This is complementary to the requirement that the same sediment (with a void ratio, the same as in the prototype) is to be used in the model for model similarity, in conjunction with Eq. (20).

Finally, we will discuss the third nondimensional soil parameter, S, Eq. (9). The model similarity requires that this quantity should have the same value in both the prototype and the model:

$$S_p = S_m \tag{24}$$

or, from Eq. (15)

$$\frac{c_{vp} T_p}{L_p^2} = \frac{c_{vm} T_m}{L_m^2} \tag{25}$$

Since the previous similarity criteria in conjunction with Eqs. (8) and (20) require that the same sediment needs to be used in the model, the coefficient of consolidation in the prototype and in the model should be the same, and therefore the similarity condition in Eq. (25) will reduce to

$$\frac{T_p}{L_p^2} = \frac{T_m}{L_m^2}, \quad \text{or} \tag{26}$$

using Eq. (3),

$$T_p = N^2 T_m \tag{27}$$

Now, considering potential model wave periods T_m, to be calculated using Eqs. (16)-(18), and the potential prototype wave periods, T_p, and the realistic values of the length scale N, it is observed that it is highly likely that the similarity condition given in Eq. (27) would never be satisfied, as $T_p = O(10s)$, $T_m = O(1s)$ and $N = O(10^3 - 10^4)$.

When inspected closely, it is seen that the nondimensional parameter in Eq. (9), S, stems from the diffusion term in the governing equation describing the buildup of pore pressure, Eq. 3.28 in Sumer (2014), the first term on the right hand side of the equation. This term becomes relatively important only towards the end of the buildup of pore pressure, and therefore may be considered to have only little effect on the end result as far as the process of buildup of pore pressure is concerned. Nevertheless, it must be borne in mind that this similarity condition will not be truly satisfied although we know that the end effect will be insignificant.

4.4. Discussion of 3D case

We have, in the preceding, developed model criteria for physical modelling of buildup of pore pressure. For simplicity, this was done for a 2D structure. The entire theory developed above for the 2D case is equally valid for a 3D structure provided that the model size in the third dimension is scaled down by the same length scale, namely, N. This is elaborated in the following paragraphs.

As seen in the discussion in Section 2, the key point for a true similarity is to simulate in the model the "interplay" between the wave length, L, and the structure width, W. As pointed out therein, to achieve this, the prototype wave length should be scaled down with the same length scale as the width of the structure, namely, N.

In the case of a 3D structure, the structure has a third dimension in the direction perpendicular to the picture plane, Fig. 1. The progressive wave at hand, however, remains the same, namely the 2D (long-crested), progressive wave. As long as the previously mentioned "interplay" between the wave length, L, and the structure width, W, is maintained in the model (by scaling down the prototype wave length by the length scale N), the buildup of pore pressure under the aforementioned 2D waves will be simulated correctly in the model, regardless whether or not the structure is 2D or 3D.

In the case of a 3D structure subjected to, not a long-crested wave but a short-crested wave with wave lengths different in two directions, L_x and L_y, with the structure dimensions W_x and W_y in plan view, the previously mentioned interplay between the wave length and the structure width needs to extend to the third direction, the direction perpendicular to the picture plane in Fig. 1. Nevertheless, the length scales in both directions can, for practical reasons, be selected to be the same, and therefore the similarity criteria developed for the 2D case above will be equally applicable in this case, too.

4.5. Nondimensional accumulated pore pressure

4.5.1. Undisturbed case

From the physics of the buildup of pore pressure, the accumulated pore pressure, p, scales with the initial mean normal effective stress, σ_0', Sumer, 2014, Section 3.1

$$\sigma_0' = \gamma' z \frac{1+2k_0}{3} \qquad (28)$$

with k_0 being the coefficient of lateral earth pressure. (The fact that the accumulated pore pressure, p, scales with σ_0' is "confirmed" with the analytical solution of Sumer and Cheng (1999), reproduced in Sumer, 2014, Eqs. 3.41-3.43.) Therefore, the similarity condition for the pore pressure can be expressed as follows, i.e., the nondimensional pore pressure $\frac{p}{\sigma_0'}$ should have the same value in both the prototype and the model:

$$(\frac{p}{\sigma_0'})_p = (\frac{p}{\sigma_0'})_m \qquad (29)$$

With the correct design of the model structure, the model wave and the model soil, the model pressure measured in the soil in the model experiments can then be translated into the prototype through Eq. (29). To this end, considering that the same soil is used in the model as in the prototype (therefore, k_0 and γ' $(=\gamma(s-1)/(1+e)$ for saturated soils) being the same in both the model and the prototype), the accumulated pore pressure in the prototype from Eq. (29) will be

$$p_p = \frac{z_p}{z_m} p_m = N p_m \qquad (30)$$

4.5.2. Case with weight of structure considered as surcharge

In the case when the weight of the structure is considered as a surcharge, then the the initial mean normal effective stress, σ_0', will be as

$$\sigma_0' = (\gamma' z + p_s)\frac{1+2k_0}{3} \qquad (31)$$

in which p_s is the surface loading (or the surcharge), Sumer, 2014, p. 127, corresponding to the weight of the structure (cf. Eqs. (28) and (31)). (We note that, in the case of not-too-shallow soils, p_s should be replaced by αp_s where α is a factor related to the spreading of the loaded area with the soil depth. See the discussion in Sumer, 2014, pp. 131-132.)

Now, for model similarity, the nondimensional pore pressure $\frac{p}{\sigma_0'}$ should have the same value in both the prototype and the model:

$$(\frac{p}{\sigma_0'})_p = (\frac{p}{\sigma_0'})_m \qquad (32)$$

in which σ_0' is, this time, given by Eq. (31). From Eq. (32) (along with Eq. (31)),

$$\frac{p_p}{\gamma_p' z_p + p_{sp}} = \frac{p_m}{\gamma_m' z_m + p_{sm}} \qquad (33)$$

Solving p_p from the above equation, and considering that the same sediment is used in the model as in the prototype (and therefore $\gamma_p' = \gamma_m' = \gamma'$),

$$p_p = p_m \frac{\gamma' z_p + p_{sp}}{\gamma' z_m + p_{sm}} \qquad (34)$$

Eq. (34) reduces to Eq. (30) when $p_{sp} = p_{sm} = 0$, the undisturbed case.

Now, if the model surcharge, p_{sm}, is selected to be

$$p_{sm} = \frac{p_{sp}}{N} \qquad (35)$$

Then the accumulated pore pressure in the prototype will, from Eq. (34), be

$$p_p = p_m \frac{\gamma' z_p + p_{sp}}{\gamma' z_m + p_{sm}} = p_m \frac{\gamma'(z_m N) + p_{sm} N}{\gamma' z_m + p_{sm}} = N p_m \quad (36)$$

Meaning that the similarity condition will be the same as in the undisturbed case, Eq. (30). This is obviously on the condition that the model surcharge is to be selected so as to satisfy Eq. (35).

4.5.3. Space and time variation

Clearly, the pore pressure in the model experiments is measured as a function of space (z in 1D case, x, z in 2D case, and x, y, z in 3D case) and time (t).

First, consider the 1D case. The similarity conditions associated with space and time are, from Eqs. (4) and (5):

$$\lambda_p z_p = \lambda_m z_m \quad (37)$$

$$\omega_p t_p = \omega_m t_m \quad (38)$$

From Eq. (37), the depth at which the pore pressure is measured in the model is translated into the prototype by

$$z_p = N z_m \quad (39)$$

In the general 3D case (see the discussion in Section 4.4), the above equation will read

$$(x_p, y_p, z_p) = N(x_m, y_m, z_m) \quad (40)$$

in which (x_m, y_m, z_m) are the coordinates of the point where the accumulated pore pressure is measured in the model, and (x_p, y_p, z_p) are the coordinates of the corresponding point in the prototype.

Regarding the similarity with respect to time, This is governed by Eq. (38). From the latter equation, the time in prototype will be

$$t_p = \frac{T_p}{T_m} t_m \qquad (41)$$

in which T_m is the wave period in the model, and T_p that in the prototype. The latter equation is used to convert the model times to the prototype times.

One final note is the following. The ratio of time and the wave period is actually equal to the number of waves, i.e., Number of Waves $= t / T$. When we consider the number of waves to cause liquefaction, this will read

$$N_{\ell p} = \frac{t_p}{T_p} \qquad (42)$$

is the number of waves to cause liquefaction in the prototype, with t_p being the time at which liquefaction occurs in the prototype, and similarly,

$$N_{\ell m} = \frac{t_m}{T_m} \qquad (43)$$

is that in the model. Now, from the similarity condition in Eq. (41):

$$N_{\ell p} = N_{\ell m} \qquad (44)$$

This implies that the number of waves to cause liquefaction will be the same in both the model and the prototype.

5. A numerical example

5.1. Prototype data

The prototype data are given as follows.

The prototype gravity foundation has a width of $W_p = 45$ m, and a height of $h_{ap} = 7.5$ m (Fig. 1). The dimension of this gravity structure perpendicular to the picture plane, W_{yp}, is assumed to be the same as W_p, namely, $W_{yp} = 45$ m. The structure is made of steel, with a density of $\rho_{steel} \approx 7.5$ tons/m^3, mass of the structure is around 14000 tons. This gives a submerged specific weight of $\gamma_{steel} \approx 63.7$ kN/m^3, and a prototype surcharge of $p_{sp} = 58.8$ kPa.

The prototype waves have the following properties: The wave height $H_p = 7.7$ m, the wave period $T_p = 9.1$ s, the water depth $h_p = 60$ m, and the wave length $L_p = 128.6$ m.

The prototype soil has a relative density of $D_{rp} = 0.28$ while the soil depth is $d_p = 10$ m.

5.2. Model data

5.2.1. Model structure

Considering the dimensions of the experimental flume to be used in the model experiments, we select the length scale as

$$N = 75 \tag{45}$$

Therefore, the structure dimensions in the model will be as follows:

$$W_m = \frac{W_p}{N} = \frac{45}{75} = 0.60 \text{ m} \tag{46}$$

$$h_{am} = \frac{h_{ap}}{N} = \frac{7.5}{75} = 0.10 \text{ m} \tag{47}$$

$$W_{ym} = \frac{W_{yp}}{N} = \frac{45}{75} = 0.60 \text{ m} \tag{48}$$

Finally, the model surcharge should be scaled such as:

$$p_{sm} = \frac{p_{sp}}{75} = 784 \text{ Pa} \tag{49}$$

5.2.2. Model wave data

We start off with the wave length. The model wave length will, from Eqs. (3) and (45), be:

$$L_m = \frac{L_p}{N} = \frac{128.6}{75} = 1.71 \text{ m} \tag{50}$$

Second, the water depth in the model, h_m, can be selected freely, considering the height of the model structure, h_{am}, and also the depth of the flume to be used in the model experiments, as discussed in detail Section 4.2. In a medium size flume facility in the laboratory, the range depicted in Table 2 covers realistic values of model water depth h_m.

Third, given L_m and h_m, the model wave period, T_m, can be determined, using Eqs. (16)-(18). The results are summarized in Table 2.

Table 2. Model wave periods for a range of selected model water depths.

Model water depth, h_m (m)	Model wave period, T_m (s)
0.3	1.17
0.4	1.10
0.5	1.07
0.6	1.06

Fourth, given the prototype wave length and water depth (L_p, h_p), and similarly, the model wave length and water depth (L_m, h_m), the model wave height H_m associated with the prototype wave height $H_p = 7.7$ can be determined from Eq. (21). The results are summarized in Table 3.

Table 3. Model wave heights for a range of selected model water depths.

Model water depth, h_m (m)	Model wave height, H_m (cm)
0.3	2.6
0.4	3.6
0.5	5.0
0.6	7.1

5.2.3. Model soil data

The soil can be selected conveniently the same as that of the prototype (see the discussion in Sections 4.2 and 4.3). Therefore, the model soil relative density is to be the same as that of the prototype, namely

$$D_{rm} = D_{rp} = 0.28 \tag{51}$$

We note that one should ensure that other soil properties in the model should be the same as those of the prototype. See the discussion in conjunction with Eq. (20) discussed in Section 4.2.

Finally, the soil depth in the model is determined from Eq. (23):

$$d_m = \frac{d_p}{N} = \frac{20}{75} = 0.13 \text{ m} \tag{52}$$

for the prototype soil depth $d_p = 10$ m (see the prototype soil data, Section 5.1). The model soil depth d_m will be 0.27 m and 0.40 m for the prototype soil depth d_p of 20 m and 30 m, respectively.

6. Conclusions

This report presents the results of a study, investigating the similarity criteria for physical modelling of buildup of pore pressure in marine soils with the presence of a gravity foundation where a scale model of the gravity foundation, a representation (or a copy) of the actual structure, is employed.

The study discusses the similarity criteria for physical modelling of the buildup of pore pressure in the soil around this gravity foundation.

First of all, the study implies that physical modelling of the previously mentioned process is feasible.

Second, given the geometrical scale of the modelling, the design of the model foundation structure is achieved in a straightforward manner, Section 4.1.

Likewise, the model wave length is designed through the same geometrical scale, Section 4.2.

Given the latter information, and utilizing the similarity conditions derived in the study based on the nondimensional parameters governing the buildup of pore pressure (Table 1), the model wave is designed, with the model wave period, the model wave height, and the model water depth determined, Section 4.2.

Likewise, the model soil can also be designed in a straightforward manner, based on the similarity conditions derived in the study, Section 4.3.

The results are supported with a numerical example, Section 5.

7. Acknowledgement

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8. References

1. NuLIMAS (2020-2023): Numerical modelling of liquefaction around marine structures. research program, funded through the ERA-NET Cofund MarTERA Program (Grant No. 728053) under EU Horizon 2020 Framework. http://nulimas.info/.

2. Sumer, B.M. and Cheng, N.-S. (1999): "A random-walk model for porepressure accumulation in marine soils". Proceedings of the 9th International Offshore and Polar Engineering Conference, ISOPE-99, Brest, France, 30. May-4. June, 1999, vol. 1, pp. 521-528.

3. Sumer, B.M. (2014): Liquefaction Around Marine Structures. World Scientific.