

SEA - STRAIT FLOW
WITH SPECIAL REFERENCE TO BOSPHORUS

By

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SEA-STRAIT FLOW WITH SPECIAL REFERENCE TO BOSPHORUS.

By B. Mutlu Sumer¹ and Mehmet Bakioğlu²

INTRODUCTION

A sea strait is a channel connecting two basins of different properties where one usually finds a two-layer current system, the layers flowing in opposite directions (e.g. the Bosphorus, the strait of Gibraltar and others).

A proper understanding of the dynamics of sea-strait flow has gained considerable importance recently associated with navigation, environmental pollution, waste-water discharge and fishing problems.

Classical information about sea-strait currents can be found in Defant (5). Assaf and Hecht(3) appear to be the first investigators who developed a two-layer dynamical model for sea straits. The reader is referred to Assaf and Hecht's work also for more recent references. Anati et al.(2) studied laboratory models of sea straits, which gives a good insight into the current pattern in a strait, despite the qualitative character of this latter work.

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A two-layer model is adopted also in this study; the dynamical equations are integrated to determine the flow rates and depths, with particular application to the Bosphorus.

GENERAL CONSIDERATIONS

Statement of problem

We shall give the statement of the problem through one particular example, namely the Bosphorus, although similar arguments hold true for other sea straits.

The Bosphorus is a "long" strait connecting the Black Sea and the Sea of Marmara (Fig.1). There is a two-layer current system in the Bosphorus: a) the upper layer flows from the Black Sea to the Sea of Marmara caused by the decline in surface elevation, h , between the Black-Sea and Sea-of-Marmara ends of the Bosphorus (Fig.2) and b) the lower layer flows in the opposite direction caused by the density difference, $\Delta\rho$, which is due to the difference in salinities.

The specific problem in this study is to determine the flow rates and the layer depths when the quantities h and $\Delta\rho$ are given.

Formulation

In a strait, one usually finds an intermediate layer between the upper and lower layers. However, as the so-called densimetric Froude number is less than unity, a sharp stable density gradient will exist in the intermediate layer and therefore can be considered as an interface between the two layers. It is this approach, namely the "two layer" model, which is adopted in this paper for the determination of the sea-strait flow.

The energy equations for the upper and lower layers respectively are

$$\frac{dh}{dx} + \frac{dh}{dx} + \frac{d}{dx} \left(\frac{V^2}{2g} \right) - \frac{f_i}{8gh_1} (V_1+V_2)^2 - \sum_j k_{(j)} \frac{V^2}{2g} \delta(x-x_j) = 0 \quad (1)$$

$$\left(1 - \frac{\Delta\rho}{\rho}\right) \frac{dh}{dx} + \frac{dh}{dx} + \frac{d}{dx} \left(\frac{V^2}{2g} \right) + \frac{f_o}{8gh_2} V_2^2 + \frac{f_i}{8gh_2} (V_1+V_2)^2 + \sum_j k_{(j)} \frac{V^2}{2g} \delta(x-x_j) = 0 \quad (2)$$

in which h_1 and h_2 = the depths of the upper and lower layers, respectively; V_1 and V_2 = the vertically averaged flow velocities; f_o and f_i = the friction factors defined by

$$\tau_o = \rho \frac{f_o}{8} V_2^2 \quad (3)$$

$$\tau_i = \rho \frac{f_i}{8} (V_1+V_2)^2 \quad (4)$$

in which τ_o and τ_i = the bottom and the interfacial shear stresses, respectively (see Fig.2). In Eqs.1 and 2, $k_{(j)}$ = the loss coefficient associated with local head losses such as those in bends, expansions etc.; x_j = the x-coordinate where the local loss occurs; and $\delta(x-x_j)$ = the Dirac delta function.

The equations of continuity, neglecting entrainment between the layers, are

$$\frac{d}{dx} (h_1 V_1) = 0 \quad (5)$$

$$\frac{d}{dx} (h_2 V_2) = 0 \quad (6)$$

The boundary conditions are given as in the following. From the geometrical considerations (Fig.2),

$$x = 0 \quad ; \quad h_1 + h_2 = D \quad (7)$$

$$x = L \quad ; \quad h_1 + h_2 = h + D \quad (8)$$

and from the energy considerations

$$x=0 \quad \text{and} \quad L; \quad F = \frac{V_1^2}{\frac{\Delta\rho}{\rho} g h_1} + \frac{V_2^2}{\frac{\Delta\rho}{\rho} g h_2} = 1 \quad (9), (10)$$

in which the latter conditions (i.e. the sum of the densimetric Froude numbers, F , is equal to unity at $x=0$ and L) imply that critical flows occur at sections $x=0$ and $x=L$, which are nothing but abrupt expansions. It should be noted that Anati et al.(2), in a laboratory model of sea strait, observed that F indeed be equal to unity at the ends while $F < 1$ in the interior of the model strait.

SOLUTION AND ANALYSIS WITH REFERENCE TO BOSPHORUS

General

Integrating Eqs. 1 and 2 from $x=0$ to $x=L$ gives

$$\frac{V^2(L)}{2g} + h_1(L) + h_2(L) = \frac{V^2(0)}{2g} + h_1(0) + h_2(0) + \frac{f_i}{8g} \int_0^L \frac{(V_1 + V_2)^2}{h_1} dx + \sum_j k(j) \frac{V^2(x_j)}{2g} \quad (11)$$

$$\frac{V^2(0)}{2g} + (1 - \frac{\Delta\rho}{\rho})h_1(0) + h_2(0) = \frac{V^2(L)}{2g} + (1 - \frac{\Delta\rho}{\rho})h_1(L) + h_2(L) + \frac{f_o}{8g} \int_0^L \frac{V_2^2}{h_2} dx + \frac{f_i}{8g} \int_0^L \frac{(V_1 + V_2)^2}{h_2} dx + \sum_j k(j) \frac{V^2(x_j)}{2g} \quad (12)$$

From Eqs. 5 and 6, one gets

$$h_1 V_1 = q_1 \quad (13)$$

$$h_2 V_2 = q_2 \quad (14)$$

in which q_1 and q_2 = the flow rates per unit width for the upper and lower layers, respectively.

To proceed further, we make the following assumption : the interface has a linear shape. Fig.3 justifies this assumption where the

shape of the Bosphorus interface is plotted against the longitudinal distance. (Note that the interface here has been determined through the density sections presented in Ref.4, transversely averaging the density sections to yield density profiles in the vertical). Anati et al.'s(2) laboratory observations support the above assumption, as they have observed an essentially linear shape of the interface except very near the ends of the model strait.

Substituting Eqs. 13 and 14 into Eqs. 11 and 12 carrying out the integrals in the latter equations (keeping in mind the fact that the depths h_1 and h_2 linearly vary in x in accordance with the assumption made in the preceding paragraph), Eqs.11 and 12 become

$$\begin{aligned}
 & h_{1L}^{-h_{10}+h_{2L}-h_{20}} + \frac{q_1^{*2}}{2} \left(\frac{1}{h_{1L}^2} - \frac{1}{h_{10}^2} \right) - \frac{f_1 L^*}{8} \left\{ \frac{q_1^{*2} (h_{10}+h_{1L})}{2h_{10}^2 h_{1L}^2} + \right. \\
 & + \frac{2q_1^* q_2^*}{h_{10}h_{2L}-h_{20}h_{1L}} \left(\frac{h_{10}-h_{1L}}{h_{10} h_{1L}} + \frac{h_{2L}-h_{20}}{h_{10}h_{2L}-h_{20}h_{1L}} \ln \frac{h_{10}h_{2L}}{h_{20}h_{1L}} \right) + \\
 & + \frac{q_2^{*2}}{h_{10}h_{2L}-h_{20}h_{1L}} \left(\frac{h_{2L}-h_{20}}{h_{2L} h_{20}} + \frac{h_{1L}-h_{10}}{h_{10}h_{2L}-h_{20}h_{1L}} \ln \frac{h_{20}h_{1L}}{h_{10}h_{2L}} \right) \} - \\
 & - \frac{q_1^{*2}}{2} \frac{(h_{1L}-h_{10})^2}{(h_{1L}-h_{10})^2} = 0 \quad (15) \\
 & - \frac{q_2^{*2}}{2} \frac{(h_{2L}-h_{20})^2}{(h_{2L}-h_{20})^2} = 0 \quad (15) \\
 & \left(1 - \frac{\Delta\rho}{\rho} \right) (h_{1L}-h_{10}) + h_{2L}-h_{20} + \frac{q_1^{*2}}{2} \left(\frac{1}{h_{1L}^2} - \frac{1}{h_{10}^2} \right) + \\
 & + \frac{f_1 L^*}{8} \left\{ \left(1 + \frac{f_0}{f_1} \right) \frac{q_1^{*2} (h_{20}+h_{2L})}{2h_{20}^2 h_{2L}^2} + \right. \\
 & + \frac{2q_1^* q_2^*}{h_{20}h_{1L}-h_{10}h_{2L}} \left(\frac{h_{20}-h_{2L}}{h_{20} h_{2L}} + \frac{h_{1L}-h_{10}}{h_{20}h_{1L}-h_{10}h_{2L}} \ln \frac{h_{20}h_{1L}}{h_{10}h_{2L}} \right) + \\
 & \left. + \frac{q_2^{*2}}{2} \frac{(h_{2L}-h_{20})^2}{(h_{2L}-h_{20})^2} \right\} = 0
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{q_1^{*2}}{h_{20} h_{1L} - h_{10} h_{2L}} \left(\frac{h_{1L} - h_{10}}{h_{1L} h_{10}} + \frac{h_{2L} - h_{20}}{h_{20} h_{1L} - h_{10} h_{2L}} \ln \frac{h_{10} h_{2L}}{h_{20} h_{1L}} \right) + \\
 & + \frac{k_2}{2} \frac{q_2^{*2}}{(h_{2L} - h_{20})^2} \left(\ln \frac{h_{2L}}{h_{20}} \right)^2 = 0 \quad (16)
 \end{aligned}$$

in which the nondimensional quantities $h_{10}, h_{1L}, h_{20}, h_{2L}, q_1^*, q_2^*, L^*$ are defined as

$$\begin{aligned}
 h_{10} &= \frac{h(0)}{D}, \quad h_{1L} = \frac{h(L)}{D}, \quad h_{20} = \frac{h_2(0)}{D}, \quad h_{2L} = \frac{h_2(L)}{D} \\
 q_1^{*2} &= \frac{q_1^2}{gD^3}, \quad q_2^{*2} = \frac{q_2^2}{gD^3}, \quad L^* = \frac{L}{D}, \quad h^* = \frac{h}{D}
 \end{aligned} \quad (17)$$

Note that, in Eqs. 11 and 12, the velocities $V_1(x_j)$ and $V_2(x_j)$ in the local-loss terms are taken as their length-averaged values, and therefore k_1 and k_2 in Eqs. 15 and 16 are the total loss coefficients for the upper and lower layers respectively :

$$\begin{aligned}
 k_1 &= \sum_j k(j) \quad \text{and} \quad k_2 = \sum_j k(j) \\
 \text{(upper)} & & \text{(lower)}
 \end{aligned} \quad (18), (19)$$

The boundary conditions (Eqs. 7-10) in terms of the new nondimensional variables are

$$h_{10} + h_{20} = 1 \quad (20)$$

$$h_{1L} + h_{2L} = 1 + h^* \quad (21)$$

$$\frac{q_1^{*2}}{h_{10}^3} + \frac{q_2^{*2}}{h_{20}^3} = \frac{\Delta\rho}{\rho} \quad (22)$$

$$\frac{q_1^{*2}}{h_{1L}^3} + \frac{q_2^{*2}}{h_{2L}^3} = \frac{\Delta\rho}{\rho} \quad (23)$$

in which h^* = the decline in surface elevation between the two ends of the Bosphorus nondimensionalized by D as given in Eq. 17.

We have six equations, Eqs. 15, 16, 20-23, and six unknowns, $h_{10}, h_{1L},$

h_{20}, h_{2L}, q_1^* and q_2^* . This set of equations was reduced to that of two non-linear algebraic equations with two unknowns, and solved by standard methods for the Bosphorus. Information about the input variables used and the routine adopted in the computations are given in Appendix I.

Solution and analysis

The interface obtained for the average decline in the surface elevation between the two ends of the Bosphorus, $h=33$ cm, is illustrated in Fig.3. The agreement between the present finding and the field data appears to be fairly good.

Also in Fig.3 is given Assaf and Hecht's (3) solution. Assaf and Hecht took into consideration, the salt-balance equation for what they called the closed basin (which is the Black Sea in our case) in addition to the momentum and continuity equations (Eqs.1,2,5 and 6 in this paper). From these equations, they arrived at a single equation an iterative solution of which, together with the boundary conditions (Eqs.9 and 10), gave the depth of interface and the "strait salinity ratio" $(S_2 - S_1)/S_1$; Note that this scheme included a term like $Q_1 - Q_2$, which was considered as a known quantity. Here Q_1 and Q_2 = the flow rates for the upper and lower layers, respectively. The interface plotted in Fig.4 is that obtained for $Q_1 - Q_2 = 6000 \text{ m}^3/\text{s}$. The value $Q_1 - Q_2 = 12000 - 6000 = 6000 \text{ m}^3/\text{s}$ is due to Moeller's analysis (Ref.4) and has long been questioned owing to the averaging procedure involved. Yet the U.S.Navy Oceanographic Office data (quoted in Ref.4) indicate that flows in either direction vary between approximately 3000 and 30000 m^3/s . The departure of Assaf and Hecht's interface from that obtained in the present study can be attributed to the somewhat erroneous value of $Q_1 - Q_2, 6000 \text{ m}^3/\text{s}$, adopted in Assaf and Hecht's solution.

Again for the average h , 33 cm, the flow rates of the upper and lower layers respectively were found to be $Q_1=19\ 700\ m^3/s$ and $Q_2=7300\ m^3/s$. These figures compare favourably well with $Q_1=17400\ m^3/s$ and $Q_2=7900\ m^3/s$ obtained from the simultaneous solution of the mass-balance and salt-balance equations for the Black Sea

$$Q_1 - Q_2 - Q_3 + Q_4 - Q_5 = 0 \quad (24)$$

$$Q_1 S_1 - Q_2 S_2 = 0 \quad (25)$$

in which Q_3 = the average runoff to the Black Sea, $352\ km^3/year$ (Ref.7); Q_4 = the average evaporation, $353\ km^3/year$ (Ref.7); Q_5 = the average precipitation $300\ km^3/year$; S_2 and S_1 = the salinities at the midpoint of the Bosphorus, $\% 38.5$ and $\% 17.5$, respectively (Ref.4). Note that Ref.7 gives average precipitations measured at meteorological stations located along the coastline of the Black Sea; from these measurements, the overall average precipitation was found to be $71.4\ cm$, which led to $Q_5=300\ km^3/year$ when multiplied with $420\ 000\ km^2$ (= the surface area of the Black Sea).

In Fig.4 is given the Froude number, F (see Eq.9), plotted against the distance x ; the finding of Fig.4 is in complete agreement with Anati et al.'s (2) laboratory observation that $F=1$ at the two ends of the strait while $F<1$ in the interior.

Further Analysis

As is reported in Appendix I, available sea-level records yielded an average decline of 33cm in surface elevation between the two ends of the Bosphorus, with a standard deviation of 13 cm. Thus it would be of interest to study the effect of h on the interface and the flow rates. The solutions obtained for various h s are presented in Figs.5 and 6, which

indicate that tremendous changes in the flow properties would occur even with moderate changes in h .

The findings of Figs.5 and 6 can also be interpreted as in the following. If any value of h (occurring in the Bosphorus) is maintained persistently for a sufficiently long period of time and yet the response of the two-layer system to changes in h is sufficiently quick, then the solutions in Figs.5 and 6 will be the exact steady-state solutions corresponding to this particular h . Assuming the latter is true, then one would expect that, from Fig.6, the lower layer will stop and behave like a "salt" wedge as $h \rightarrow 45$ cm, while this will be the case for the upper layer as $h \rightarrow 10$ cm, the upper layer behaving like a "temperature" wedge in this case. However rough the above assumption is (which led to the preceding result), the latter is significant in the sense that it shows the effect of h upon the mechanism which drives the two-layer current system in sea straits. Finally, it should be noted that the flow rates in Fig.6 are in accord with the range 3000-30000 m^3/s reported by the U.S. Navy Oceanographic Office quoted in Ref.(4).

As a final remark, it would be interesting to note that, for the average value of the decline in surface elevation $h=33$ cm, no sediment transport has been observed, as the Shields parameter in this case was found to be less than the threshold value. However, for lesser values of h (i.e. for greater lower-layer discharges), the Shields parameter takes values of around 0.05 with the corresponding grain Reynolds number of 30-40, which indicates that a weak sediment-transport stage may occur in the Bosphorus.

CONCLUSIONS

A two-layer model is adopted to determine the average properties of

sea-strait flows. The model neglects the wind stress on the surface because of its short-term character. It also does not take into consideration the side-wall effects as it assumes very large aspect ratios (e.g. the latter is about 1:30 for the Bosphorus).

The model has been solved for the Bosphorus with the assumption that the interface has a linear shape. The interface thus obtained (Fig.3) appeared to be in fairly good agreement with the field data. Through the solution of the model, also the flow rates of the upper and lower layers have been obtained; $Q_1=19700 \text{ m}^3/\text{s}$ and $Q_2=7300 \text{ m}^3/\text{s}$. These figures were found to be in accord with those obtained from the equations of mass and salt conservations for the Black Sea. Despite the uncertainties involved in the numerical values of the various input quantities due to insufficient data (particularly that in the decline in surface elevation between the two ends of the Bosphorus) and also the assumption in connection with the shape of the interface, the results seem to be satisfactory when compared with the available information. We believe that the model can successfully be used for other straits as well.

One of the advantages of the model is that it predicts the flow rates occurring in straits, the direct measurement of which usually involves practical difficulties because of extremely large cross-sections, the lack of proper instrumentation, the constant presence of navigation etc.

Finally, the authors would like to point out the considerable influence of the decline in surface elevation between the two ends of the strait, h , upon the flow. This implies that changes in h throughout

the year (depending on various effects such as wind, runoff etc.) will result in drastic changes in the two-layer current system. In this connection, one poses the following questions: What is the duration, on the average, for the decline h to be persistent? How does the two-layer system respond to changes in h , and how long does it take for the two-layer system to reach a new state of equilibrium? How does the latter period of time compare with the time duration during which h is persistent? Time-dependent solutions are needed to answer these questions, which, in author's opinion, are of great importance as to the dynamics of sea straits.

ACKNOWLEDGEMENT

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APPENDIX I.-INPUT AND ROUTINE IN COMPUTATIONS

The set of equations, Eqs.15,16,20-23, was solved for the Bosphorus. In what follows information about the input employed and the routine adopted in the computations are given.

Input variables

These include the density difference, the geometry and the decline in surface elevation between the two ends of the Bosphorus. These properties are given in Table:1 where the figures are the average values of the corresponding quantities. For the average values of the depth and width, averaging has been achieved through seven cross-sections out of those given in (Ref.4, Plate A IV 1 and Fig.A III 1); These particular cross-sections were chosen such that a) their locations are evenly

distributed along the Bosphorus and b) the sections themselves are located out of the separation zones. This made it possible to get non-biased average values for the relevant quantities.

As to the decline in surface elevations between the two ends of the Bosphorus, Ref.4 (Plate A IV 20) presents continuous recordings of average daily sea surface elevations at stations located in Kavak and Üsküdar (see Fig.1), corresponding to the period July 1966 to February 1968; Through the latter data, an average decline of 33 cm with a standart deviation of 13 cm was found, extrapolating the figures obtained for the reach Kavak-Üsküdar to the whole length of the Bosphorus.

Loss coefficients

Contributions of various effects to the total loss coefficient k_1 were estimated to be as in the following: total contribution to k_1 due to bends= 2.2, that due to expansions and contractions= 2.1 , that due to entry =0.5 and that due to exit= 1.0 ; thus the overall sum gives $k_1 \approx 6$.

The total loss coefficient for the lower layer k_2 was taken to be equal to k_1 , since, from the bathymetry of Bosphorus (Ref.4, Fig.A III 1), it has been observed that the bends and the expansions and contractions of the lower layer appear to be reasonably consistent with those of the upper layer.

Friction factors

To predict the bottom friction factor f_o , the following expressions (Ref.6) were used (whichever is applicable) :

$$\frac{1}{\sqrt{f_o}} = 2 \log_{10} \left(\frac{Re\sqrt{f_o}}{2.51} \right) \quad \text{when} \quad \frac{k_s u_*}{\nu} < 5 \quad (26)$$

$$\frac{1}{\sqrt{f_o}} = -2 \log_{10} \left(\frac{k_s}{12R} + \frac{2.5}{\text{Re}\sqrt{f_o}} \right) \quad \text{when } 5 < \frac{k_s u_*}{\nu} < 70 \quad (27)$$

$$\frac{1}{\sqrt{f_o}} = 2 \log_{10} \left(\frac{12R}{k_s} \right) \quad \text{when } \frac{k_s u_*}{\nu} > 70 \quad (28)$$

in which u_* = the bottom shear velocity; k_s = the equivalent sand roughness which can be taken as d_{65} of the bottom material as recommended by Raudkivi (8). In the preceding equations

$$\text{Re} = \frac{4V R}{\nu} \quad (29)$$

in which V is taken as the length-averaged value of the lower-layer velocity V_2 , and $R = h_2/2$, h_2 here being the length-averaged value of the lower-layer depth. Note that the friction factor predicted in the course of computations appears to be in the range of 0.009-0.011.

As far as the interfacial friction factor f_i is concerned, Abraham et al.'s (1) "lock-exchange" data were adopted for the present purpose; the latter data were approximated to

$$f_i = \frac{0.5}{\text{Re}^{0.45}} \quad \text{when } 10^2 < \text{Re} < 2 \times 10^4 \quad (30)$$

$$f_i = 0.0056 \quad \text{when } \text{Re} > 2 \times 10^4 \quad (31)$$

in which Re is defined as in the preceding paragraph. However, the interfacial friction factor predicted in the course of computations appeared to be 0.0056, the asymptotic value, as the Reynolds number came out to be as $\text{Re} > 2 \times 10^4$

Routine

In what follows, the iteration procedure used in the computations is summarized: a) Assign some value to f_o and also that to f_i ; b) Solve the set of Eqs. 15, 16, 20-23 to determine the layer depths and velocities; c) Compute the Re number; d) Calculate f_o and f_i , using

Eqs.26-31 ; e) Continue this procedure until the differences between the new values of f_0 and f_i and the previous ones are judged to be small enough.

APPENDIX II. -REFERENCES

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APPENDIX III.- NOTATION

The following symbols are used in this paper :

- D = depth of the strait ;
- f_i = interfacial friction factor ;
- f_o = bottom friction factor ;
- g = acceleration due to gravity ;
- h = decline in surface elevation between the two ends of the strait ;
- h^* = h non dimensionalized by D .
- h_1 = upper-layer depth ;
- h_2 = lower-layer depth ;
- $h_1(0), h_1(L)$ = depths of upper layer (interface) at the two ends of the strait ;
- $h_2(0), h_2(L)$ = depths of lower layer at the two ends of the strait ;
- h_{10}, h_{1L} = depths of upper layer at the two ends of the strait, nondimensionalized by D ;
- h_{20}, h_{2L} = depths of lower layer at the two ends of the strait, nondimensionalized by D ;
- $k_{(j)}$ = loss coefficient associated with local head loss ;
- k_1 = total loss coefficient for the upper layer ;
- k_2 = total loss coefficient for the lower layer ;
- L = length of the strait ;
- L^* = length of the strait nondimensionalized by D ;
- Q_1 = upper-layer flow rate ;
- Q_2 = lower-layer flow rate ;
- q_1 = upper-layer flow rate per unit width ;
- q_2 = lower-layer flow rate per unit width ;
- q_1^* = upper-layer flow rate per unit width nondimensionalized as

$$q_1^{2*} = q_1^2 / (g D^3) \quad ;$$

q_2^* = lower-layer flow rate per unit width nondimensionalized as

$$q_2^{2*} = q_2^2 / (g D^3) \quad ;$$

V_1 = upper-layer velocity ;

V_2 = lower-layer velocity ;

x = longitudinal distance from one end of the strait (longitudinal distance from the Sea-of-Marmara end with reference to the Bosphorus) ;

ρ = density ;

$\Delta\rho$ = density difference ;

τ_i = interfacial shear stress; and

τ_o = bottom shear stress ;

TABLE 1. - Input variables for the Bosphorus .

Density difference $\frac{\Delta\rho}{\rho}$	Length L, in kilometers	Depth D, in meters	Width, in meters	Decline in surface elevation between two ends, in centimeters
(1)	(2)	(3)	(4)	(5)
0.014	31.25	64.5	907	33

Information Retrieval Abstract

SEA-STRAIT FLOW WITH SPECIAL REFERENCE TO BOSPHORUS

KEY WORDS : Black Sea, Bosphorus, Hydrodynamics, Oceanography ,
Sea straits, Stratified flow, Straits, Two-layer current system,
Waterways

ABSTRACT : A two-layer model (comprising one energy equation and one continuity equation for the upper layer and those for the lower layer) is adopted. The model takes into account both the interfacial and the bottom frictions. Under the assumption that the interface has a linear shape, the equations are integrated to yield the layer depths and the flow rates. Numerical results obtained for the Bosphorus, the strait connecting the Black Sea and the Sea of Marmara. The interface predicted by the present model appears to be in fairly good agreement with the available field data. The flow rates predicted are in accord with those obtained from the equations of mass and salt conservations for the Black Sea. It was found that the decline in surface elevation between the two ends of the Bosphorus has a significant influence on the two-layer current system occurring in the Bosphorus.

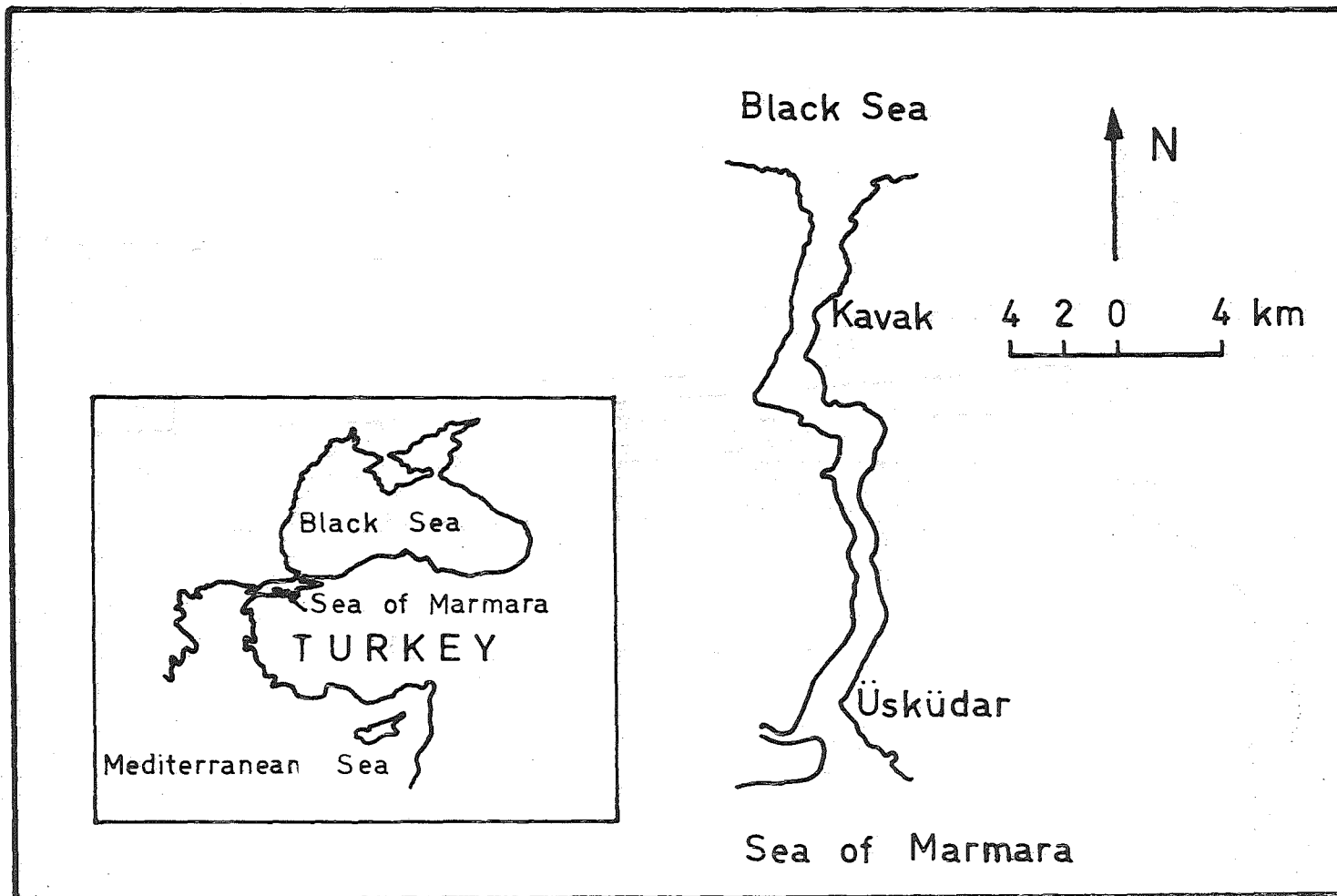


Fig.1- The Bosphorus

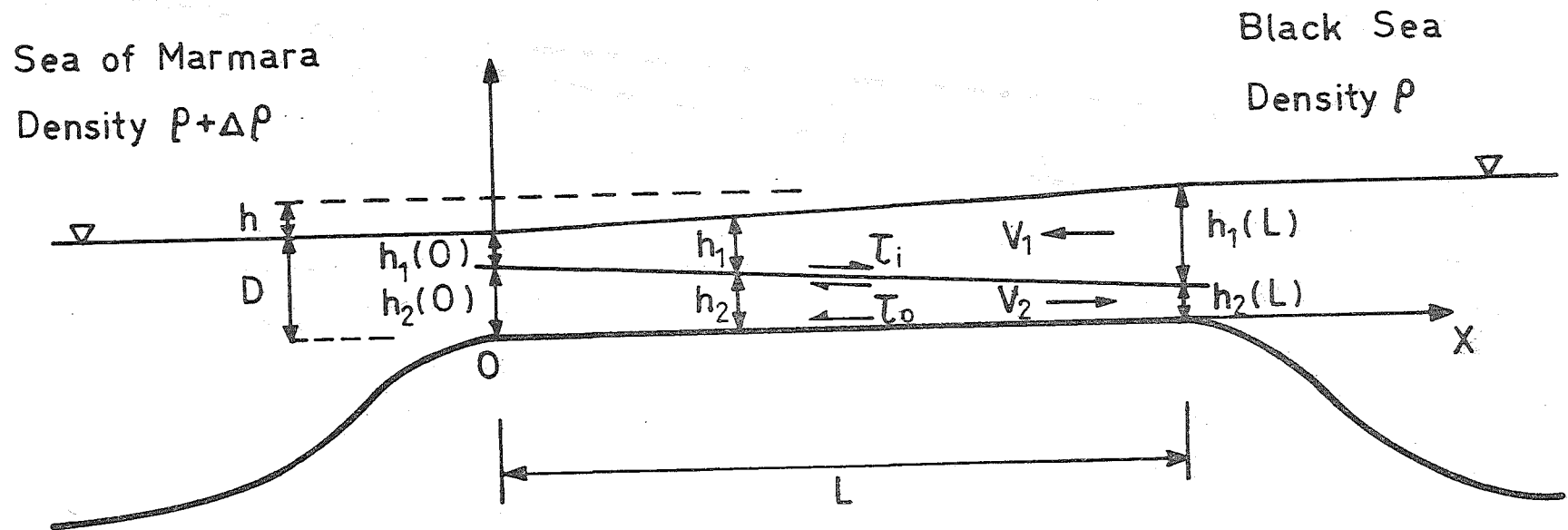


Fig.2- Definition sketch

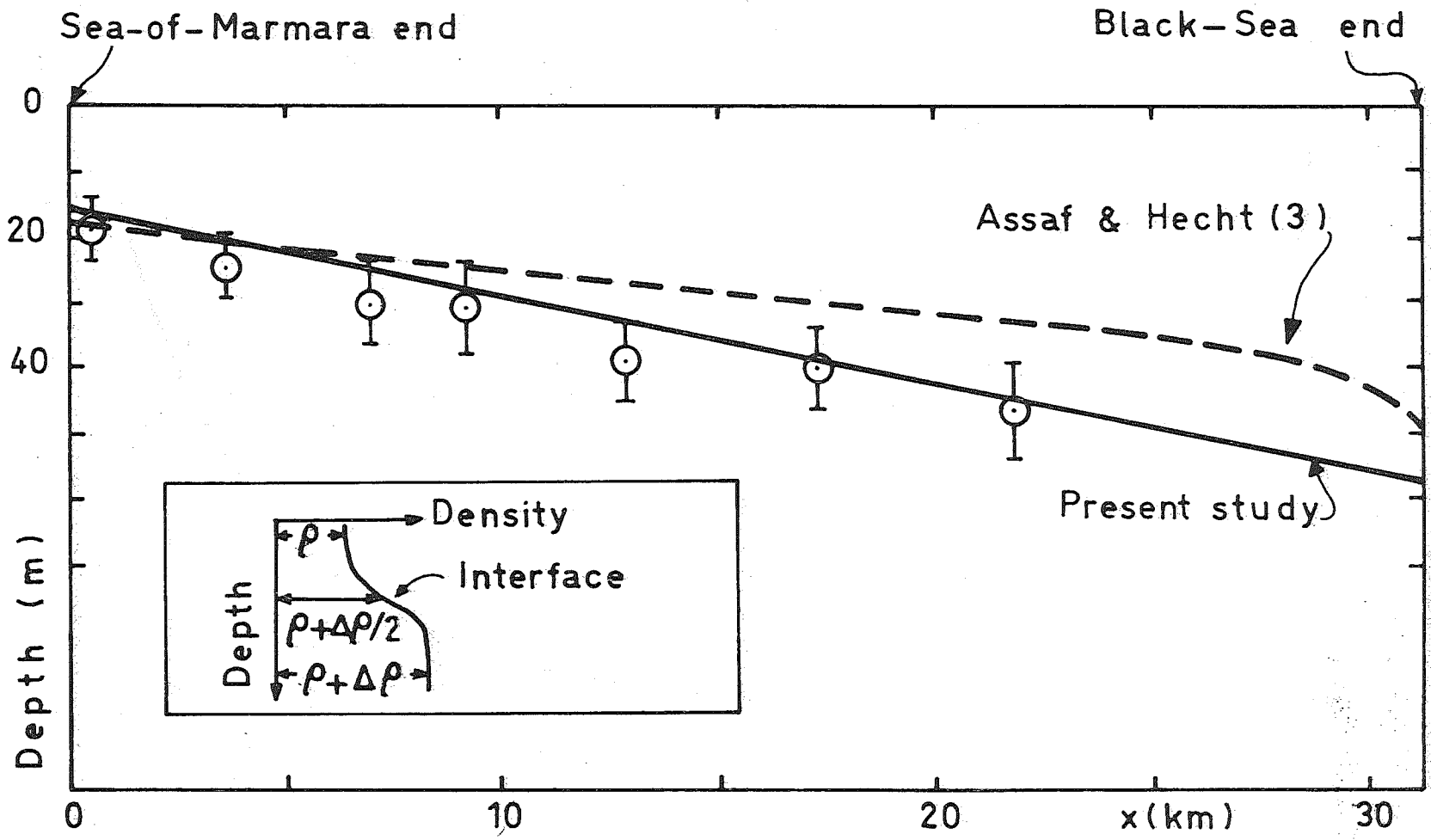


Fig.3- Interface in the Bosphorus. Data comprise the density observations made on 23-29 March, 2-5 April, 29 June-3 July, 7-11 Sept., 13-15 Oct., 12-18 Nov. in 1965 and 5-8 Jan., 11-16 Feb. in 1966 reported in Ref.(4, Plates A IV 10-A IV 13). I denotes two times standart deviation. *June* $h=33$ cm.

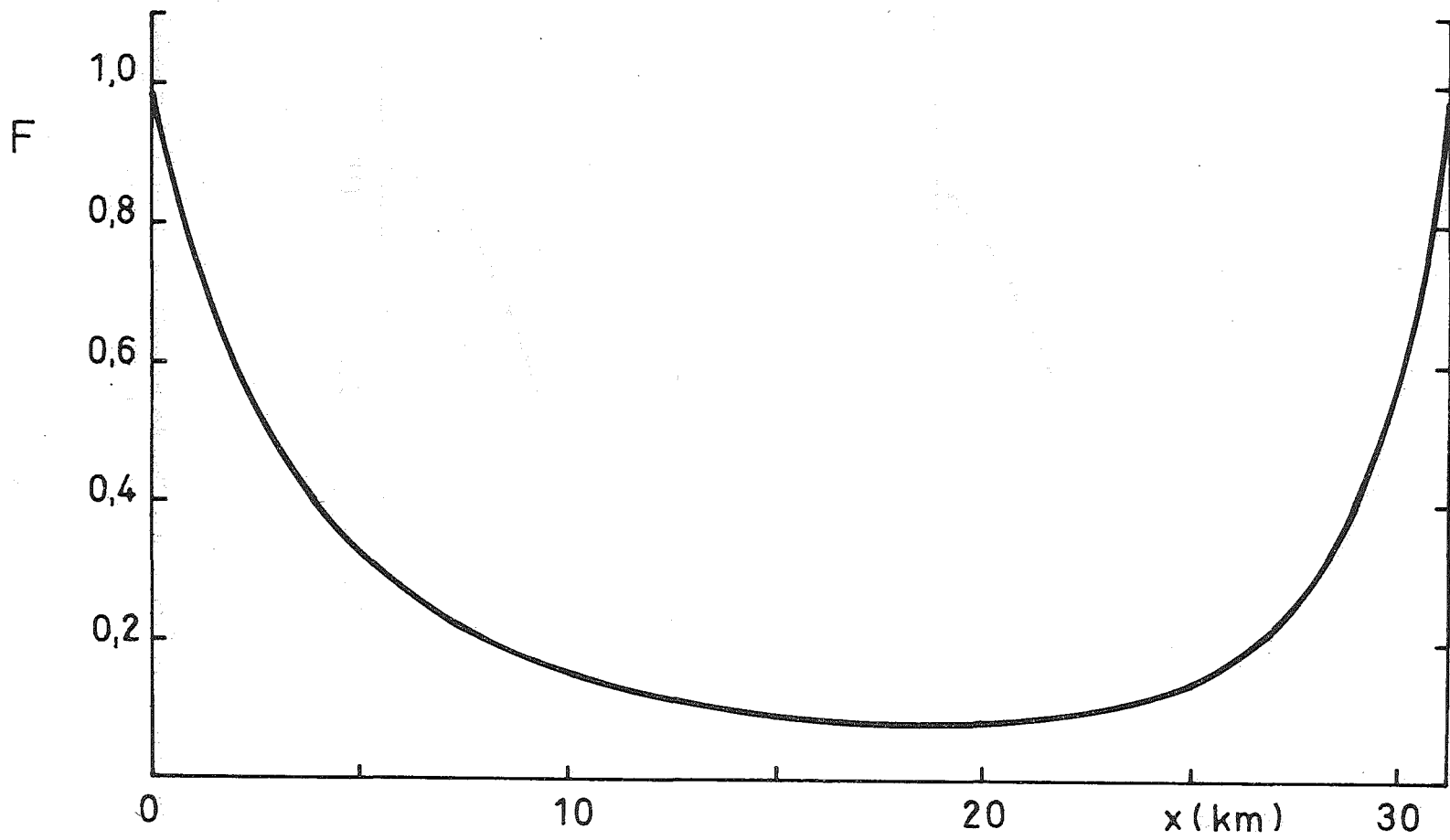


Fig.4 - Froude number $F = \frac{v_1^2}{\left(\frac{\Delta\rho}{\rho} g h_1\right) + v_2^2 / \left(\frac{\Delta\rho}{\rho} g h_2\right)}$ versus distance from the Sea-of-Marmara end. $h = 33$ cm.

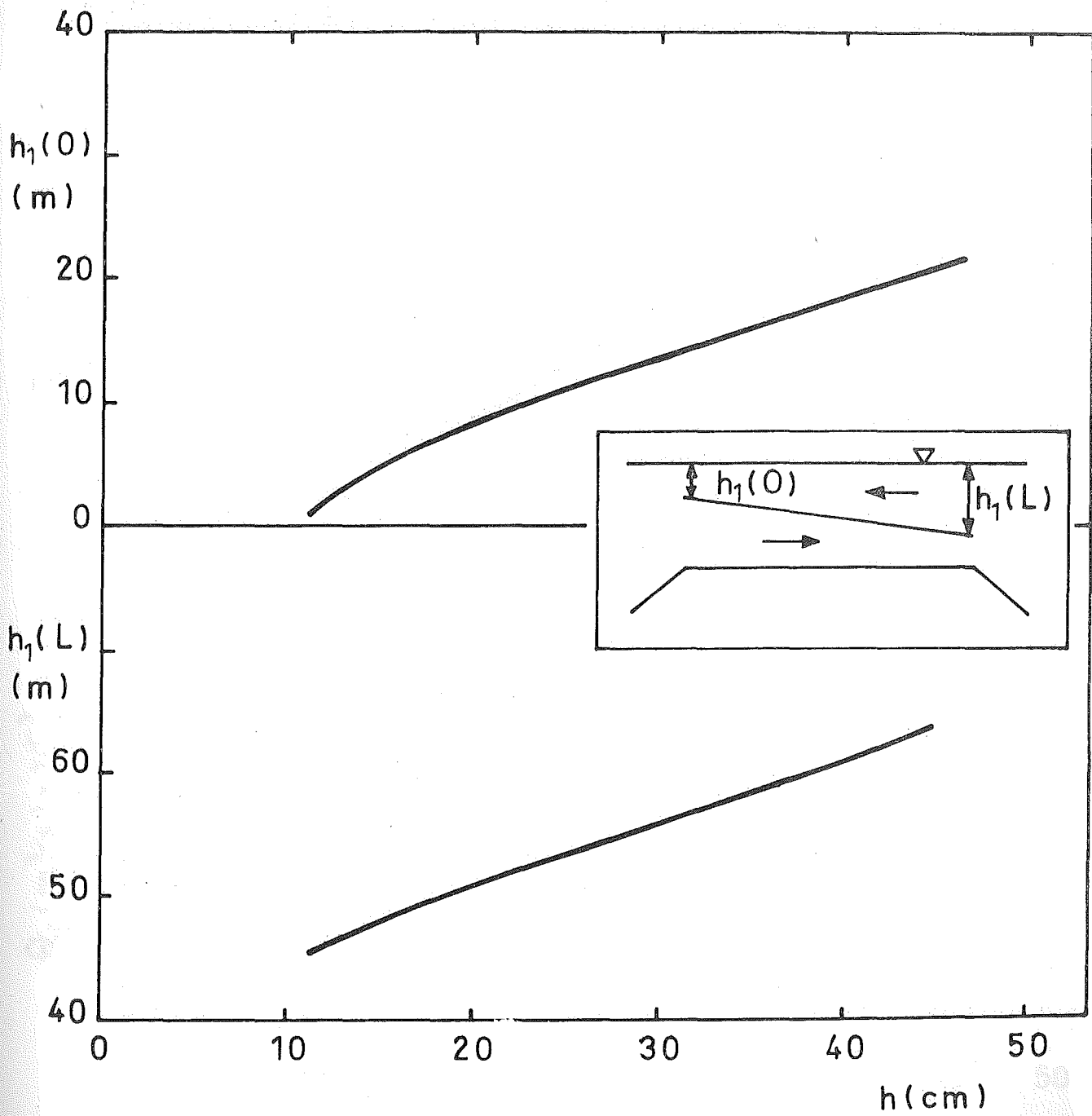


Fig.5 - Depths of interface at the two ends as a function of decline in surface elevation between these two ends of the Bosphorus.

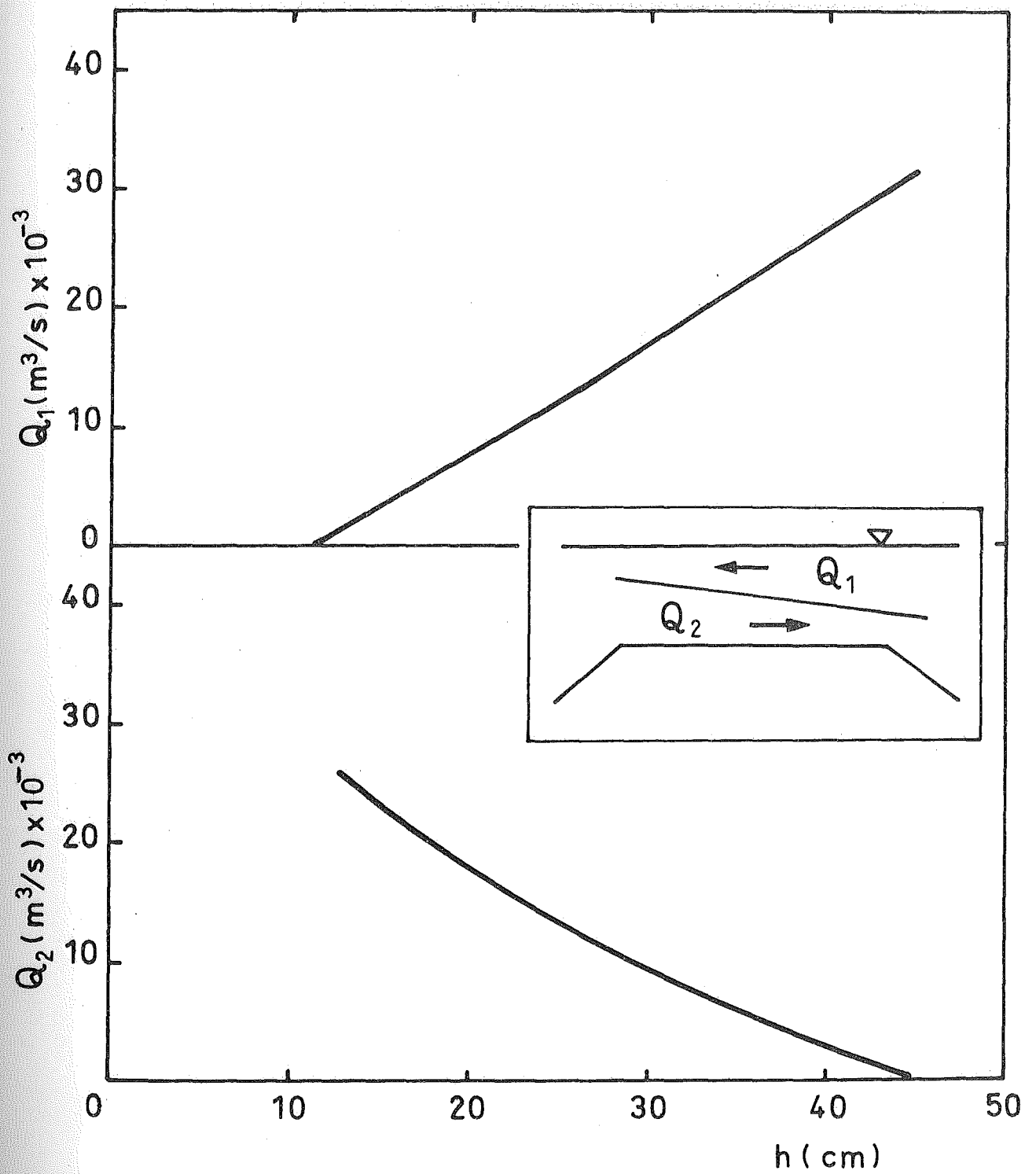


Fig.6- Flow rates of the layers as a function of decline in surface elevation between the two ends of the Bosphorus.